

Trends in Mathematical And Data Sciences

**June, 16th - 20th, 2025
BRAZIL**

Book of Abstracts

Trends in Mathematical And Data Sciences

Organizing committee

Enrique Zuazua (FAU - Germany) - Chair

Juan Límaco (FAU - Brazil) - Chair

Sandra Malta (LNCC - Brazil)

Mauro Rincon (UFRJ - Brazil)

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José Bautista (UFF - Brazil)

José Rodríguez (UFF - Brazil)

Laurent Prouvee (UERJ - Brazil)

Suerlan Silva (UFF - Brazil)

Welcome

It is with great enthusiasm that we announce the "Trends in Mathematical and Data Science" workshop, which will take place from June 16 to 20, 2025, at the Institute of Mathematics and Statistics of the Universidade Federal Fluminense, in Niterói, Rio de Janeiro, Brazil.

This event is part of the PROBRAL-CAPES-DAAD project, a collaboration between the Universidade Federal Fluminense (coordinated by Prof. Juan Limaco) and the Friedrich-Alexander-Universität Erlangen-Nürnberg (coordinated by Prof. Enrique Zuazua).

This international workshop will bring together researchers from FAU-Germany and Brazil, fostering the growing connection between academia and industry. Tools from Machine Learning, Partial and Ordinary Differential Equations, and Control Theory will be used to explore the increasing application of mathematical models to industrial problems.

During the event, we will have exciting sessions, such as:

Digital Twins: Where Mathematics and Industry Meet Control Methods and Numerical Analysis of Complex Systems One of the main goals of this workshop is to strengthen the ties between mathematicians from Friedrich-Alexander-Universität and Brazilian researchers, as well as to promote new partnerships between FAU and academic and research institutions in Brazil.

The Organizing Committee

TRENDS IN MATHEMATICAL AND DATA SCIENCES TMADS 25 WORKSHOP

from June 16th to 20th 2025
Niteroi - Rio de Janeiro, Brazil



ORGANIZING COMMITTEE

Enrique Zuazua FAU, Germany (Chair)
Juan Limaco UFF, Brazil (Chair)
Sandra Malta LNCC (Brazil)
Mauro Rincon UFRJ (Brazil)
Carlos Guzman UFF (Brazil)

LOCAL ORGANIZING COMMITTEE

Cristian Loli (UFF) Laurent Prouvee (UERJ)
Jose Bautista (UFF) Gabriel Correa (UFF)
Suerlan Silva (UFF) Andre Rocha (UERJ)
Jose Rodriguez (UFF) Ricardo Fuentes (UFF)

Projeto: PROBAL-CAPES-DAAD, UFF-FAU



<http://grupoedp.uff.br>



DAAD



Friedrich - Alexander - Universität
DYNAMICS, CONTROL,
MACHINE LEARNING,
AND NUMERICS



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Conference Schedule

Monday, June 16		
Time (UTC-3)	Speakers	Title
11:00–12:20	Morning Session zoom	
11:40–12:20	Juan Limaco, Javier Ribon, Ana Maria Farias, Monica Savedra, Enrique Zuazua	Opening Ceremony
12:20–14:30	Lunch	
14:30–16:30	Afternoon Session zoom	
14:30–15:10	Liliane Basso Barichello (UFRGS)	Concise and Accurate Discrete Ordinates Solutions of the Neutral Particle Transport Equation
15:10–15:50	Alexandre Madureira (LNCC)	Numerical Methods for High Contrast PDEs
15:50–16:30	Paulo Amorim (FGV)	Predator-prey and epidemiological models using transport equations
Tuesday, June 17		
08:30–12:20	Morning Session zoom	
08:30–09:10	Nuno Crokidakis (UFF)	From smoke to simulations: Understanding the papal conclave dynamics through agent-based models
09:10–09:50	Liviu Ignat (FAU)	Approximations of the best constants
09:50–10:30	Sergey Sergeev (UFRJ)	The homogenization procedure for the localized waves propagation
10:30–11:00	Coffee Break	
11:00–11:40	Maria Soledad Aronna (FGV)	Optimal Control of under Parametric Uncertainty
11:40–12:20	Round Table	
12:20–14:30	Lunch	
14:30–16:30	Afternoon Session zoom	
14:30–15:10	Lorenzo Liverani (FAU)	Stability of hyperbolic systems with non-symmetric relaxation
15:10–15:50	Sergey Tikhomirov (PUC-RIO)	Estimate mixing zone in miscible displacement in porous media
15:50–16:30	Felipe Linares (IMPA)	On decay and asymptotic properties of solutions to the Intermediate Long Wave equation

Wednesday, June 18		
08:30–12:20	Morning Session zoom	
08:30–09:10	Jose Cuminato (USP)	Data Science Center for Smart Industry (CDII)
09:10–09:50	Tiago Roux Oliveira (UERJ)	Extremum Seeking Boundary Control: From Wave Dynamics with Kelvin-Voigt Damping to the Stefan Problem
09:50–10:30	Marcio Murad (LNCC)	Innovative scheme for multiphase flow in deformable porous media
10:30–11:00	Coffee Break	
11:00–11:40	Pablo Blanco (LNCC)	Towards real-life computational hemodynamics
11:40–12:20	Round Table	
12:20–14:30	Lunch	
14:30–16:30	Afternoon Session zoom	
14:30–15:10	Yuri Saporito (FGV)	Functional Itô Calculus: Theory, applications and numerical methods
15:10–15:50	Amaury Alvarez Cruz (UFRJ)	Generalized Solutions in Geochemical Models: A Colombeau Algebra Approach
15:50–16:30	Max Souza (UFF)	A mathematical framework for dynamical social interactions with dissimulation
Thursday, June 19		
08:30–12:20	Morning Session zoom	
08:30–09:10	José Rodríguez Ayllón (UFF)	Spatio-temporal data with cylindrical shearlets
09:10–09:50	Jaqueline Godoy Mesquita (UNB)	Volterra-Stieltjes functional integral equations and applications
09:50–10:30	Daniel Alfaro (UFRJ)	Numerical analysis of the Schrödinger equation in a moving domain.
10:30–11:00	Coffee Break	
11:00–11:40	Poster Session 01	Jose Bautista, Miguel Soto, Orlando Romero, Suerlan Silva and Oscar Sierra
11:40–12:20	Poster Session 02	Alfredo Soliz, Camila Kalil, Cristian Loli and Fredson Aguiar
12:20–14:30	Lunch	
14:00–16:20	Afternoon Session zoom	
14:00–14:40	Boyan Sirakov (PUC-RIO)	Elliptic regularity estimates with optimized constants and applications

14:50–15:30	Wladimir Neves (UFRJ)	Stability of planar rarefaction waves
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Global Time Zone Schedule and Zoom link

Event	Rio (UTC-3)	Chicago (UTC-5)	London (UTC+1)
June 16 - Monday			
Morning Session - Join Zoom zoom			
Opening Ceremony	11:40-12:20	09:40-10:20	15:40-16:20
Lunch	12:20-14:30	10:20-12:30	16:20-18:30
Afternoon Session - Join Zoom zoom			
Lecture 1	14:30-15:10	12:30-13:10	18:30-19:10
Lecture 2	15:10-15:50	13:10-13:50	19:10-19:50
Lecture 3	15:50-16:30	13:50-14:30	19:50-20:30
June 17 - Tuesday			
Morning Session - Join Zoom zoom			
Lecture 1	08:30-09:10	06:30-07:10	12:30-13:10
Lecture 2	09:10-09:50	07:10-07:50	13:10-13:50
Lecture 3	09:50-10:30	07:50-08:30	13:50-14:30
Coffee Break	10:30-11:00	08:30-09:00	14:30-15:00
Lecture 4	11:00-11:40	09:00-09:40	15:00-15:40
Round Table	11:40-12:20	09:40-10:20	15:40-16:20
Lunch	12:20-14:30	10:20-12:30	16:20-18:30
Afternoon Session - Join Zoom zoom			
Lecture 5	14:30-15:10	12:30-13:10	18:30-19:10
Lecture 6	15:10-15:50	13:10-13:50	19:10-19:50
Lecture 7	15:50-16:30	13:50-14:30	19:50-20:30
June 18 - Wednesday			
Morning Session - Join Zoom zoom			
Lecture 1	08:30-09:10	06:30-07:10	12:30-13:10
Lecture 2	09:10-09:50	07:10-07:50	13:10-13:50
Lecture 3	09:50-10:30	07:50-08:30	13:50-14:30
Coffee Break	10:30-11:00	08:30-09:00	14:30-15:00
Lecture 4	11:00-11:40	09:00-09:40	15:00-15:40
Round Table	11:40-12:20	09:40-10:20	15:40-16:20
Lunch	12:20-14:30	10:20-12:30	16:20-18:30
Afternoon Session - Join Zoom zoom			
Lecture 5	14:30-15:10	12:30-13:10	18:30-19:10
Lecture 6	15:10-15:50	13:10-13:50	19:10-19:50
Lecture 7	15:50-16:30	13:50-14:30	19:50-20:30

June 19 - Thursday			
Morning Session - Join Zoom zoom			
Lecture 1	08:30-09:10	06:30-07:10	12:30-13:10
Lecture 2	09:10-09:50	07:10-07:50	13:10-13:50
Lecture 3	09:50-10:30	07:50-08:30	13:50-14:30
Coffee Break	10:30-11:00	08:30-09:00	14:30-15:00
Poster Session 1	11:00-11:40	09:00-09:40	15:00-15:40
Poster Session 2	11:40-12:20	09:40-10:20	15:40-16:20
Lunch	12:20-14:30	10:20-12:30	16:20-18:30
Afternoon Session - Join Zoom zoom			
Lecture 5	14:00-14:40	12:00-12:40	18:00-18:40
Lecture 6	14:50-15:30	12:50-13:30	18:50-19:30
Lecture 7	15:40-16:20	13:40-14:20	19:40-20:20
June 20 - Friday			
Morning Session - Join Zoom zoom			
Lecture 1	08:30-09:10	06:30-07:10	12:30-13:10
Lecture 2	09:10-09:50	07:10-07:50	13:10-13:50
Lecture 3	09:50-10:30	07:50-08:30	13:50-14:30
Coffee Break	10:30-11:00	08:30-09:00	14:30-15:00
Lecture 4	11:00-11:40	09:00-09:40	15:00-15:40
Closing Ceremony	11:40-12:20	09:40-10:20	15:40-16:20
Lunch	12:20-14:30	10:20-12:30	16:20-18:30
Afternoon Session - Join Zoom zoom			
Lecture 5	14:30-15:10	12:30-13:10	18:30-19:10
Lecture 6	15:10-15:50	13:10-13:50	19:10-19:50
Lecture 7	15:50-16:30	13:50-14:30	19:50-20:30

Event	Madrid (UTC+2)	Lima (UTC-5)	Berlin (UTC+2)
June 16 - Monday			
Morning Session - Join Zoom zoom			
Opening Ceremony	16:40-17:20	09:40-10:20	16:40-17:20
Lunch	17:20-19:30	10:20-12:30	17:20-19:30
Afternoon Session - Join Zoom zoom			
Lecture 1	19:30-20:10	12:30-13:10	19:30-20:10
Lecture 2	20:10-20:50	13:10-13:50	20:10-20:50
Lecture 3	20:50-21:30	13:50-14:30	20:50-21:30

June 17 - Tuesday			
Morning Session - Join Zoom zoom			
Lecture 1	13:30-14:10	06:30-07:10	13:30-14:10
Lecture 2	14:10-14:50	07:10-07:50	14:10-14:50
Lecture 3	14:50-15:30	07:50-08:30	14:50-15:30
Coffee Break	15:30-16:00	08:30-09:00	15:30-16:00
Lecture 4	16:00-16:40	09:00-09:40	16:00-16:40
Round Table	16:40-17:20	09:40-10:20	16:40-17:20
Lunch	17:20-19:30	10:20-12:30	17:20-19:30
Afternoon Session - Join Zoom zoom			
Lecture 5	19:30-20:10	12:30-13:10	19:30-20:10
Lecture 6	20:10-20:50	13:10-13:50	20:10-20:50
Lecture 7	20:50-21:30	13:50-14:30	20:50-21:30
June 18 - Wednesday			
Morning Session - Join Zoom zoom			
Lecture 1	13:30-14:10	06:30-07:10	13:30-14:10
Lecture 2	14:10-14:50	07:10-07:50	14:10-14:50
Lecture 3	14:50-15:30	07:50-08:30	14:50-15:30
Coffee Break	15:30-16:00	08:30-09:00	15:30-16:00
Lecture 4	16:00-16:40	09:00-09:40	16:00-16:40
Round Table	16:40-17:20	09:40-10:20	16:40-17:20
Lunch	17:20-19:30	10:20-12:30	17:20-19:30
Afternoon Session - Join Zoom zoom			
Lecture 5	19:30-20:10	12:30-13:10	19:30-20:10
Lecture 6	20:10-20:50	13:10-13:50	20:10-20:50
Lecture 7	20:50-21:30	13:50-14:30	20:50-21:30
June 19 - Thursday			
Morning Session - Join Zoom zoom			
Lecture 1	13:30-14:10	06:30-07:10	13:30-14:10
Lecture 2	14:10-14:50	07:10-07:50	14:10-14:50
Lecture 3	14:50-15:30	07:50-08:30	14:50-15:30
Coffee Break	15:30-16:00	08:30-09:00	15:30-16:00
Poster Session 1	16:00-16:40	09:00-09:40	16:00-16:40
Poster Session 2	16:40-17:20	09:40-10:20	16:40-17:20
Lunch	17:20-19:30	10:20-12:30	17:20-19:30
Afternoon Session - Join Zoom zoom			

Lecture 5	19:00-19:40	12:00-12:40	19:00-19:40
Lecture 6	19:50-20:30	12:50-13:30	19:50-20:30
Lecture 7	20:40-21:20	13:40-14:20	20:40-21:20

Plenary Talks

NUMERICAL ANALYSIS OF THE SCHRÖDINGER EQUATION IN A MOVING DOMAIN.

DANIEL G. ALFARO VIGO ¹

In this talk, we present the mathematical analysis of the convergence of the linearized Crank-Nicolson Galerkin method for a nonlinear Schrödinger problem related to a domain with a moving boundary. We carry out the convergence analysis of the numerical method for both semidiscrete and fully discrete problems and establish an optimal error estimate in the L^2 -norm. We provide numerical simulations to confirm the consistency of these theoretical results.

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GENERALIZED SOLUTIONS FOR NONLINEAR WAVE INTERACTIONS IN
GEOCHEMICAL MODELS: A COLOMBEAU ALGEBRA APPROACH

AMAURY ALVAREZ CRUZ ²

This work extends the analysis of nonlinear conservation laws in geochemical flows by constructing generalized solutions for systems with complex surface chemical interactions. Building on the wave curve method developed in the original article - which solved Riemann problems through composite waves, shocks, and resonance surfaces - we formalize these solutions within the framework of Colombeau algebras.

Classical weak solutions (in the Lax sense) are embedded into generalized function spaces, enabling the handling of strong nonlinearities and singularities arising from ion-rock surface interactions (coefficient terms). Rankine-Hugoniot conditions and wave bifurcations at inflection loci are rigorously interpreted using distributional products, ensuring compatibility with the geometric structure of characteristic speeds.

Numerical case studies demonstrate how this approach resolves discontinuous initial data in low-salinity carbonated water injection scenarios, typical in oil recovery. Structural stability under perturbations of coefficient terms is proven, preserving local hyperbolicity and wave speed hierarchies. The results unify analytical and numerical methods, offering a robust tool for reactive flows in heterogeneous porous media.

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PREDATOR-PREY AND EPIDEMIOLOGICAL MODELS USING TRANSPORT EQUATIONS

PAULO VERDASCA AMORIM ³

We present, analyse and simulate models for predator-prey interaction and epidemiology using transport (or structured) equations. The predator-prey model consists of a nonlocal transport equation for the predator structured by predator hunger, coupled to an ODE for the prey. By taking an asymptotic regime of fast hunger variation, we find that this system provides new interpretations and derivations of several variations of the classical Lotka–Volterra system, including the Holling-type functional responses. We next establish a well-posedness result by means of a fixed-point method. Finally, we show that in the basin of attraction of the nontrivial equilibrium, the asymptotic behaviour of the original coupled PDE-ODE system is completely described by solutions of an ODE system. We introduce a second, related model, which includes ratio-dependent response functions and present some improved results. In the epidemiological model, a healthy population is structured by disease awareness and susceptibility. This gives rise to a 2d transport equation with nonlocal terms and an associated ODE system related to SIR systems, which we analyze. We show an asymptotic result, numerical experiments, and discuss some epidemiological insights from the model.

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OPTIMAL CONTROL OF UNDER PARAMETRIC UNCERTAINTY

MARIA SOLEDAD ARONNA ⁴

Our research focuses on optimal control problems with parameter uncertainty, which involve optimizing systems governed by families of controlled ordinary differential equations. These equations are parameterized on a probability space representing the range of possible parameter values. We develop necessary optimality conditions and numerical algorithms tailored to this problem class. We show numerical simulations that validate our results.

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CONCISE AND ACCURATE DISCRETE ORDINATES SOLUTIONS OF THE NEUTRAL
PARTICLE TRANSPORT EQUATION

LILIANE BASSO BARICHELLO ⁵

The transport equation, a linear version of the Boltzmann equation, is the fundamental model describing particle distribution in a material medium. Developing analytical and computational methods for solving the transport equation is essential for establishing new technologies, such as the innovative small modular nuclear reactors, or medical techniques, such as optical tomography imaging. It is a very complex integrodifferential model involving seven independent variables. Numerical simulations in this field are challenging and a source of continued research. In this talk, we discuss recent advances in the (ADO) Analytical Discrete Ordinates method for solving the multidimensional transport equation, with special attention to optical tomography problems where scattering is a relevant issue. In the case of photon transport, the equation is usually called the Radiative Transfer Equation. The formulation is derived using the exact representation of the scattering phase function. In addition, the inverse problem of reconstructing the absorption and scattering coefficients in a slab geometry is analyzed. Finally, the main features of the method are related to recent contributions in other fields of neutral particle transport, such as the computational modeling of neutron transport applied to reactor shielding problems.

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TOWARDS REAL-LIFE COMPUTATIONAL HEMODYNAMICS

PABLO JAVIER BLANCO⁶

Unveiling the role of the fundamental forces that drive human physiology and pathophysiology relies on a deep understanding of the wide range of biomechanical environments that the cardiovascular system is exposed to during our daily life, as well as of the complex interplay among circulatory phenomena taking place at different temporal and spatial scales. The lack of proper experimental settings to address questions involving wave propagation phenomena, cardiac-arterial coupling, or conditions such as hypertension, among others, established a favorable scenario for the development of systemic-scale mathematical models of circulation in the human body. While quite comprehensive lumped parameter representations have been proposed, the use of distributed parameter models (1D models) has been limited to studying the cardiovascular system under simplified physiological scenarios such as the resting state, the supine position, and occasionally disease conditions. When thinking of incrementing the realism of these cardiovascular 1D simulations, it is important to highlight that the impact of factors such as respiration, control mechanisms, gravity, and diverse physiological states (e.g., exercise or sleep conditions), has only been partially explored. Furthermore, it is noteworthy that such conditions pose several challenges that range from the model assembling process to the numerical solvers and their computational implementations. This talk aims to explore some of our latest advancements in the field of computational hemodynamics at the systemic scale exploiting an Anatomically Detailed Arterio-Venous Network 1D model (ADAVN model). Specifically, we will discuss the integration of highly detailed models of the cardiovascular system, the impact of respiration, and the simulation of blood flow in massive vascular networks, to account for microcirculatory environments.

⁶Laboratório Nacional de Computação Científica - LNCC, Petrópolis - Rio de Janeiro, email: pjblanco@lncc.br

FROM SMOKE TO SIMULATIONS: UNDERSTANDING THE PAPAL CONCLAVE
DYNAMICS THROUGH AGENT-BASED MODELS

NUNO CROKIDAKIS ⁷

This talk explores the dynamics of papal conclaves through the lens of agent based modeling. We develop two computational models to investigate how social influence, strategic voting, and ideological alignment affect the time and outcome of the election of a new pope. In the first model, cardinals interact through imitation and responsiveness to the leading candidate, while also engaging in useful voting by abandoning hopeless candidates. The second model incorporates ideological blocs, conservatives and progressives, based on realistic distributions from recent conclaves. We simulate both scenarios and show that ideological polarization can delay consensus, especially when strategic flexibility is limited. Our results reproduce the short duration of the 2025 conclave and align well with historical data from earlier conclaves, suggesting that informal consensus building, possibly even before the voting begins, plays a crucial role. This study offers insights into the dynamics of collective decision-making under strict consensus rules, with broader implications for political and organizational behavior.

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DATA SCIENCE CENTER FOR SMART INDUSTRY (CDII)

JOSÉ CUMINATO ⁸

The presentation will provide an overview of the CDII, its objectives, collaborations with industry, and its role in human resource development. The CDII is funded by FAPESP and MCTI, and is a collaboration between USP, Unicamp, Unesp, and SENAI São Paulo. Its goal is to advance the application of Data Science techniques in São Paulo's industrial sector and to increase awareness and understanding of this field among business leaders and industry stakeholders. The partnership with SENAI plays a key role in training human resources in AI and data. Selected successful projects will be briefly presented.

⁸USP - Universidade de São Paulo, São Paulo, Brazil

VOLTERRA-STIELTJES FUNCTIONAL INTEGRAL EQUATIONS AND APPLICATIONS

JAQUELINE GODOY MESQUITA⁹

In this talk, I will present recent results on Volterra-Stieltjes functional integral equations and highlight their importance in various applications. I will show how this general class of equations unifies and extends several types of classical equations, providing a versatile framework for modeling a wide range of important phenomena.

⁹Universidade de Brasília, Departamento de Matemática, Campus Universitário Darcy Ribeiro, Asa Norte, Brasília-DF, email: gmesquita@unb.br

APPROXIMATIONS OF THE BEST CONSTANTS

LIVIU IGNAT¹⁰

In this talk we consider some well known quotients related with either eigenvalue problems, Sobolev or Hardy's inequality. We consider the infimum of these quotients and their discrete analogous in a finite element subspace. We estimate the difference between the best constants above as the discretization parameter goes to zero and obtain sharp convergence rates.

¹⁰Friedrich Alexander Universität Erlangen Nürnberg - Alexander von Humboldt Professorship, Germany.
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ON DECAY AND ASYMPTOTIC PROPERTIES OF SOLUTIONS TO THE INTERMEDIATE
LONG WAVE EQUATION

FELIPE LINARES¹¹

In this talk we will consider solutions to the initial value problem associated with the intermediate long wave (ILW) equation. Our first goal will present persistence properties of the solution flow in weighted Sobolev spaces. We will show that our results are sharp. In the second part of the talk we will be concerned with the long time dynamics of large solutions to the ILW equation. Using virial techniques, we will describe regions of space where the energy of the solution must decay to zero along sequences of times. Moreover, in the case of exterior regions, we prove complete decay for any sequence of times. The remaining regions not treated here are essentially the strong dispersion and soliton regions.

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STABILITY HYPERBOLIC SYSTEMS WITH NON-SYMMETRIC RELAXATION

LORENZO LIVERANI ¹²

In this talk, I will discuss a recent work (in collaboration with T. Crin-Barat, L. Y. Shou, and E. Zuazua) concerning the stability of one-dimensional linear hyperbolic systems with non-symmetric relaxation. Introducing a new frequency-dependent Kalman stability condition, we prove an abstract decay result underpinning a form of inhomogeneous hypocoercivity. In contrast with the homogeneous setting, the decay rates depend on how the Kalman condition is fulfilled and, in most cases, a loss of derivative occurs: one must require an additional regularity assumption on the initial data to ensure the decay. Under structural assumptions, we refine our abstract result by providing an algorithm, of wide applicability, for the construction of Lyapunov functionals. This allows us to systematically establish decay estimates for a given system and uncover algebraic cancellations (beyond the reach of the Kalman-based approach), reducing the loss of derivatives in high frequencies.

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WLADIMIR NEVES¹³

In this talk, we consider the vanishing dissipation limit of the compressible Navier-Stokes-Fourier system, where the initial data approach a profile generating a planar rarefaction wave for the limit Euler system. We show that the associated weak solutions converge unconditionally to the planar rarefaction wave strongly in the energy norm.

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NUMERICAL METHODS FOR HIGH CONTRAST PDE'S

ALEXANDRE MADUREIRA ¹⁴

In this talk I'll show some special finite element methods that can handle elliptic PDEs with high-contrast coefficients. For these methods, convergence holds even for solutions with minimum regularity. The methods are based on spectral decompositions and require Caccioppoli type arguments to localize some solutions.

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AN INNOVATIVE SEQUENTIAL SCHEME FOR SOLVING MULTIPHASE FLOW IN DEFORMABLE POROUS MEDIA

MARCIO ARAB MURAD ¹⁵

We propose a novel computational framework to solve the Black-Oil model for multiphase flow in deformable porous media, incorporating geomechanical coupling through the fixed-stress split scheme. The extended flow equations, which describe the transport of two slightly compressible liquids and a highly compressible gas, are reformulated as a multiphase, multicomponent flow system. To address the coupled nature of flow and deformation, we introduce two alternative sequential coupling strategies at different levels, linking the flow/transport and mechanics subsystems. These strategies feature a consistent definition of a trusted saturation variable embedded in the flow equations, enabling full resolution of a three-equation hyperbolic system. This significantly enhances both the stability and accuracy of the proposed scheme. The flow and mechanics subsystems are discretized using mixed finite element formulations. Meanwhile, the system of conservation laws governing the phase compositions is solved using a novel semi-discrete central-upwind finite volume scheme for hyperbolic conservation laws. The proposed numerical model demonstrates a strong capability to capture the intricate interplay between geomechanical effects and phase changes, especially near the bubble point, where such interactions are most critical.

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EXTREMUM SEEKING BOUNDARY CONTROL FOR PDES: FROM WAVE DYNAMICS
WITH KELVIN-VOIGT DAMPING TO THE STEFAN PROBLEM

TIAGO ROUX OLIVEIRA¹⁶

Extremum Seeking (ES) has evolved from applications in static maps to dynamic networks, with recent advances extending its applicability to systems governed by partial differential equations (PDEs). This talk provides an overview of ES algorithm design and theory for two classes of infinite-dimensional systems: hyperbolic dynamics, such as wave PDEs with Kelvin-Voigt damping, and parabolic dynamics, including diffusion PDEs with moving boundaries related to material phase changes. Applications range from deep-sea source seeking and additive manufacturing to studies of polar ice, demonstrating the broad relevance of ES in complex and emerging engineering systems.

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EFFICIENT REPRESENTATION OF SPATIO-TEMPORAL DATA USING CYLINDRICAL SHEARLETS

JOSÉ PEDRO RODRÍGUEZ AYLLÓN¹⁷

This work introduces a multiscale directional system of functions based on cylindrical shearlets to address the challenge of representation of spatio-temporal data. This kind of data is not efficiently represented by conventional representation systems due to limitations of handling the geometry of the data; that is, the singularities which correspond in this case to the first coordinates but the last one. This kind of data typically corresponds to sequences of videos of 2 and 3 dimensional images, where the singularities lie. This work proves that this new approach achieves superior approximation properties with respect to conventional multiscale representations.

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FUNCTIONAL ITÔ CALCULUS: THEORY, APPLICATIONS AND NUMERICAL METHODS

YURI FAHAM SAVORITO ¹⁸

Functional Itô calculus is a generalization of Itô's stochastic calculus, enabling the examination of models that depend on the history of certain factors. In this talk, we will survey its theoretical development and explore some applications in Mathematical Finance. As the main application, we will demonstrate how this theory allows us to address stochastic optimal control problems with path-dependent influences of the control on the dynamics of the state process. Additionally, we will discuss numerical implementations that leverage deep learning methods to solve the functional equations arising from this theory.

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THE HOMOGENIZATION PROCEDURE FOR THE LOCALIZED WAVES PROPAGATION

SERGEY SERGEEV¹⁹

We will consider the propagation of the localized waves in the inhomogeneous media. The wave propagation is described by the initial value problem for the wave equation with variable coefficient. The initial data for the problem is considered localized. The coefficient in the equation is considered fast oscillating and therefore the homogenization procedure appears.

In this case we have two different small parameters - size of the initial conditions and parameter of the oscillations. Depending on the ratio between those two small parameters, the homogenized equation can be different. In the case of the sufficiently large, compared to the oscillations, area of the perturbation, the homogenized equation will be just the wave equation. But if the initial perturbation is not sufficiently large, then the additional coefficients will appear in the homogenized equation. These coefficients add the dispersion effects on the solution of the homogenized equation.

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ELLIPTIC REGULARITY ESTIMATES WITH OPTIMIZED CONSTANTS AND
APPLICATIONS

BOYAN SIRAKOV²⁰

We revisit the classical theory of linear second-order uniformly elliptic equations in divergence form whose solutions have Hölder continuous gradients, and prove versions of the generalized maximum principle, the $C^{1,\alpha}$ -estimate, the Hopf-Oleinik lemma, the boundary weak Harnack inequality and the differential Harnack inequality, in which the constant is optimized with respect to the norms of the coefficients of the operator and the size of the domain. Our estimates are complemented by counterexamples which show their optimality. We also give applications to the Landis conjecture and spectral estimates.

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A MATHEMATICAL FRAMEWORK FOR DYNAMICAL SOCIAL INTERACTIONS WITH
DISSIMULATION

MAX SOUZA²¹

Modeling social interactions is a challenging task that requires flexible frameworks. For instance, dissimulation and externalities are relevant features influencing such systems — elements that are often neglected in popular models. This paper is devoted to investigating general mathematical frameworks for understanding social situations where agents dissimulate, and may be sensitive to exogenous objective information. Our model comprises a population where the participants can be honest, persuasive, or conforming. Firstly, we consider a non-cooperative setting, where we establish existence, uniqueness and some properties of the Nash equilibria of the game. Secondly, we analyze a cooperative setting, identifying optimal strategies within the Pareto front. In both cases, we develop numerical algorithms allowing us to computationally assess the behavior of our models under various settings. Joint work with Y. Saporito and Y. Thamsten.

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VARIOUS METHODS TO ESTIMATE MIXING ZONE IN MISCIBLE DISPLACEMENT IN POROUS MEDIA

SERGEY TIKHOMIROV²²

Phenomenon of viscous fingers plays an important role in petroleum engineering in case of highly viscous oil or certain enhanced oil recovery methods [1, 2]. Laboratory and numerical experiments show the linear growth of the mixing zone, and we are interested in determining the exact speed of propagation of fingers. Knowledge of the precise value of the speed of the fingers would allow to optimize injection scheme of certain enhanced oil recovery methods (for instance polymer and surfactant-polymer flooding [2]). The existing theoretical upper bounds for the growth rate of the mixing zone are higher than the observed speed from the numerical simulations [4]. In the talk we present several numerical strategies to determine size of the mixing zone for homogenous and heterogeneous media [1, 3, 4] and concrete method to improve design of surfactant-polymer flooding.

For further improvements of the estimates we theoretically study gravitational fingering phenomenon - the unstable displacement of miscible liquids in porous media with the speed determined by Darcy's law. Such model is called incompressible porous medium equation (IPM). We believe that one of the possible mechanisms of slowing down the fingers' growth is due to convection in the transversal direction [4], and explain it by introducing a semi-discrete model. The model consists of a system of advection-reaction-diffusion equations on concentration, velocity and pressure in several vertical tubes (real lines) and interflow between them. In the simplest setting of two tubes we show the structure of gravitational fingers - the profile of propagation is characterized by two consecutive travelling waves which we call a terrace. We prove the existence of such a propagating terrace for the parameters corresponding to small distances between the tubes [5]. While for multiple tubes the solution has more complicated structure than propagating terrace, a structures similar to two-tubes model describe significant part of the solution.

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Poster Session

A LEARNING-BASED MODEL PREDICTIVE CONTROL FRAMEWORK FOR EPIDEMIC MITIGATION UNDER PARAMETRIC AND STATE UNCERTAINTY

FREDSON S. S. AGUIAR AND M. SOLEDAD AROMA²³

On early stages of a new infectious disease spread, little to no immunization measure is available, meaning testing and social distancing are the front-line mitigation tools available. That said, we consider an *SIR* model, augmented with a quarantine compartment representing the fraction of the quarantined population. This gives us the *SIRQ* dynamics:

$$\begin{aligned}\dot{S} &= -\beta SI, \\ \dot{I} &= \beta SI - (\gamma_1 + u)I, \\ \dot{Q} &= uI - \gamma_2 Q, \\ \dot{R} &= \gamma_1 I + \gamma_2 Q,\end{aligned}$$

where the control u represents the testing rate in that population. We consider the testing rate constraint $0 < u_{\min} \leq u(t) \leq u_{\max}$, representing a minimum required and the maximum possible testing rates, and a pure state constraint $I(t) \leq I_{\max}$, representing the intended epidemic mitigation. For the problem, we assume one has only reliable measures of the quarantined population and testing rates; we also assume only the recovering rate from quarantine γ_Q to be known.

β	infection rate general population
γ_I	recovery rate after infection
γ_Q	recovery rate after quarantine

Under the mild assumptions, we recover the infected states by considering the discretization:

$$Q_k \approx Q_{k-1} + h(u_{k-1}I_{k-1} - \gamma_Q Q_{k-1})$$

and by rearranging the expression, we get reliable measurements with a one-day time delay:

$$I_{k-1} \approx \frac{Q_k - hQ_{k-1}(1 - \gamma_1)}{hu_{k-1}},$$

what is also considered to be reliably observed.

We propose an approach in which the dynamics for the infected stage is only partially known: the control effect over the system is assumed to be known, but the control-independent dynamics or parameters are unknown. That is, given the dynamics:

$$\dot{I}(t) = f(t, I, \theta) + g(t, I, u), \quad I(t_0) = I_0,$$

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where g is known, but f or θ is unknown, and assume we observe a solution $\bar{I}(t)$ to the control-free dynamics

$$\dot{\bar{I}}(t) = f(t, \bar{I}, \theta), \quad \bar{x}(0) = I_0,$$

then we propose strategies to recover approximations to the original dynamics in order to choose a control.

We avoid the estimation of the unknown states and parameters by considering a Long Short-Term Memory (LSTM) model to obtain the control-free predictions given past infected state measurements, provided to control computation.

We obtain positive outcomes that show how this kind of approach could be a viable alternative for epidemic disease mitigation under parameter and state uncertainty.

ON THE CONTROLLABILITY OF LAMINATED TIMOSHENKO BEAMS WITH
DYNAMICAL BOUNDARY CONDITIONS

GEORGE JOSE BAUTISTA SANCHEZ²⁴

This work investigates the boundary controllability of the Timoshenko laminated beam system under dynamical boundary conditions. The controllability is established through the derivation of a suitable observability inequality for the adjoint system, which is then utilized within the framework of the well-known Hilbert Uniqueness Method (HUM) from control theory. To obtain the observability inequality, we employ a combination of techniques from semigroup theory, refined spectral analysis and by contradiction following the so-called "compactness-uniqueness" argument of Zuazua.

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NUMERICAL ANALYSIS OF A QUASILINEAR HYPERBOLIC EQUATION

CAMILA DE ANDRADE KALIL²⁵

In this paper, we perform a theoretical analysis of a quasilinear hyperbolic equation. In the theoretical part of the study, we demonstrate the existence and uniqueness of solutions for the model under investigation.

$$\begin{cases} u''(x, t) - \frac{\partial}{\partial x} \left(\hat{a}(u) \frac{\partial u}{\partial x}(x, t) \right) - \frac{\partial^2 u'}{\partial x^2}(x, t) = 0 & 0 < x < L, t > 0, \\ u(0, t) = 0, & \text{on } \Gamma_0 \times (0, \infty) \\ \hat{a}(u) \frac{\partial u}{\partial x}(x, t) + \frac{\partial u'}{\partial x}(x, t) + u''(x, t) = 0, & \text{on } \Gamma_1 \times (0, \infty) \\ u(x, 0) = u_0, \quad u'(x, 0) = u_1 & 0 < x < L \end{cases}$$

This system can be regarded as a mathematical model for small longitudinal vibrations of the cross sections of a bar of length L that is fixed at one end and has a mass $M = 1$ attached to the other end. Similar models were discussed in [3], where $u(x, t)$ denotes the displacement of the cross section x of the bar at time t , and $u' = \frac{\partial u}{\partial t}$.

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HIERARCHICAL NULL CONTROL OF LINEAR DEGENERATE PARABOLIC EQUATIONS IN MOVING DOMAINS

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This work is devoted to the analysis of a null hierarchical control strategy for a class of one-dimensional degenerate parabolic partial differential equations (PDEs) defined on time-dependent spatial domains. Such models naturally arise in various physical and biological processes involving evolving media, such as heat transfer in degradable materials, diffusion in growing biological tissues, and fluid flow in media with moving boundaries.

The system under consideration is governed by the equation:

$$u_t - (a(x)u_x)_x + c(x, t)u = \chi_{\mathcal{O}}h + \chi_{\mathcal{O}_1}v^1, \quad \text{in } (x, t) \in \hat{Q}$$

where, $\hat{Q} = \{(x, t) \in (0, \ell(t)) \times (0, T)\}$ is a non-cylindrical (moving) spatial-temporal domain, $a(x)$ is a degenerate diffusion coefficient, $\chi_{\mathcal{O}}$ and $\chi_{\mathcal{O}_1}$ are the characteristic functions of the control subdomains \mathcal{O} and \mathcal{O}_1 , respectively, $h(x, t)$ is the leader (principal) control, and $v^1(x, t)$ is the follower (secondary) control.

The control strategy is formulated within a Stackelberg game framework, where the leader control $h(x, t)$ is designed to ensure null controllability of the state variable, while anticipating the optimal response $v^1(x, t)$ of the follower. The follower control seeks to minimize a cost functional that depends on the state and control variables, subject to the leader's decision.

In the first part of the study, we analyze the emergence and structure of Nash-type equilibria in semilinear PDE systems with multiple interacting agents. The complexity of the problem is magnified by the time-dependent geometry of the domain, which introduces substantial theoretical and numerical challenges. A central contribution of this work is the derivation of a Carleman estimate tailored to the degenerate nature of the diffusion operator and the non-cylindrical domain. This inequality yields a suitable observability estimate, which forms the cornerstone for establishing null controllability results in the hierarchical setting.

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EXACT LOCAL CONTROLLABILITY BY TRAJECTORIES FOR A QUASI-LINEAR PARABOLIC PDE WITH BILINEAR-CONTROL

CRISTIAN AMADOR LOLI PRUDENCIO²⁷

This paper deals with the exact control of a quasi-linear parabolic PDE with bilinear-control, in two and three dimension, that is to say $\Omega \subset \mathbb{R}^N$, $N = 2$ or 3

$$\begin{cases} u_t - \nabla \cdot (a(u)\nabla u) + F(u) = h1_\omega u & \text{in } Q = \Omega \times (0, T) \\ u(x, t) = 0 & \text{in } \Sigma = \partial\Omega \times (0, T) \\ u(x, 0) = u_0(x) & \text{on } \Omega \end{cases}$$

The novelty in this work is the appearance of the spatial derivative of the solution instead of considering only the solution in the quasi-linear term (nonlinearity), here lies the difficulty of approaching said equation and in the second member the bilinear form. First, we study the linear problem through its adjoint problem and the associated Carleman inequality and then, we use the results obtained in the linear case to conclude the nonlinear problem by applying the Listernik's Inverse Function Theorem (see for instance [1],[2],[3],[4]).

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NULL CONTROLLABILITY OF THE COMPLETE LADYZHENS KAYA-BOUSSINESQ SYSTEM

ORLANDO NOEL ROMERO OBLITAS²⁸

We study the local null controllability of the complete Ladyzhenskaya-Boussinesq system. The approach begins with the analysis of the associated linearized system, for which we prove null controllability using a suitable Carleman inequality for the corresponding adjoint system. After establishing the required functional framework and defining the relevant operators, we apply the Local Right Inversion Theorem (also known as Ljusternik's Theorem) to extend the controllability result to the full nonlinear system.

Let $\Omega \subset \mathbb{R}^N$, ($N = 2, 3$) be a nonempty, connected, open set with a regular enough boundary Γ . Let $\omega \subset \Omega$ be a nonempty open subset, and let $T > 0$. We will use the notation $Q = \Omega \times (0, T)$ and $\Sigma = \Gamma \times (0, T)$, and the outward unit normal to Ω at the point $x \in \Gamma$ will be denoted by $n(x)$. In the present work, we will be studying the null controllability problem related to the following controlled system:

$$\begin{cases} y_t - \nabla \cdot ((\nu_0 + \nu_1 |\nabla y|^2) \nabla y) + (y \cdot \nabla) y + \nabla p = v \chi_\omega + \theta e_N & \text{in } Q, \\ \nabla \cdot y = 0 & \text{in } Q, \\ \theta_t - \nabla \cdot ((\nu_0 + \nu_1 |\nabla y|^2) \nabla \theta) + y \nabla \theta = v_0 \chi_\omega + (\nu_0 + \nu_1 |\nabla y|^2) Dy : \nabla y & \text{in } Q, \\ y = 0, \theta = 0 & \text{on } \Sigma, \\ y(\cdot, 0) = y_0, \theta(\cdot, 0) = \theta_0 & \text{in } \Omega. \end{cases} \quad (1)$$

where

$$e_N = \begin{cases} (0, 1) & \text{if } N = 2, \\ (0, 0, 1) & \text{if } N = 3, \end{cases}$$

and

$$|\nabla y| := \left(\sum_{i,j=1}^N (\partial_j y_i)^2 \right)^{1/2}$$

and

$$Dy = \frac{1}{2}(\nabla y + \nabla^T y)$$

and

$$Dy : \nabla y = \sum_{j=1}^N \sum_{i=1}^N \frac{1}{2} \left(\frac{\partial y_j}{\partial x_i} + \frac{\partial y_i}{\partial x_j} \right) \frac{\partial y_i}{\partial x_j}$$

Here, e_N stands for the gravity vector field, $y = y(x, t)$ velocity of the fluid particles, $\theta = \theta(x, t)$ temperature, (y_0, θ_0) initial states, ν_0 and ν_1 are strictly positive constants, v

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and v_0 stand for control functions that act over the system just through the control domain ω . The main result of this work is stated below.

Theorem 1. *There exists $\epsilon > 0$ such that, if $(y_0, \theta_0) \in H^5(\Omega)^N \cap V \times H^5(\Omega) \cap H_0^1$ satisfying the compatibility conditions $Ay_0, A^2y_0 \in H_0^1(\Omega)^N$, $\Delta\theta_0, \Delta^2\theta_0 \in H_0^1$, as well as $\|(y_0, \theta_0)\| \leq \epsilon$, we can find $v \in L^2(\omega \times (0, T))^N$, $v_0 \in L^2(\omega \times (0, T))$ and (y, p, θ) associated state of (1.1) satisfying $y(T) = 0$, $\theta(T) = 0$.*

The proof combines a global Carleman estimate for the adjoint system with a local inversion argument based on Liusternik's theorem. This allows us to first control the linearized system and then lift the result to the nonlinear setting under suitable smallness and regularity assumptions.

OPTIMAL FEEDBACK CONTROL OF UNCERTAIN DYNAMICAL SYSTEMS

OSCAR A. SIERRA FONSECA AND M. SOLEDAD ARONNA²⁹

We investigate optimal control problems for systems with uncertain parameters represented by probability distributions, focusing on Riemann–Stieltjes, average, and ensemble control formulations where parameter uncertainty significantly affects the dynamics. We establish necessary optimality conditions for control-affine systems with uncertain initial conditions by extending the Pontryagin Maximum Principle (PMP) within a Hilbert space framework, allowing for the analysis of infinite-dimensional parameter spaces and singular arcs. For the scalar case, we provide explicit feedback control characterizations. The theoretical framework is validated numerically using a sample average approximation method, applied to several examples, showing strong consistency between the derived solutions and computational results. Our work generalizes the classical PMP to uncertain parameter spaces, offering a rigorous foundation for ensemble control design with applications in aerospace, biology, and reinforcement learning.

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EXISTENCE OF CONTROLS INSENSITIZING THE GRADIENT OF THE SOLUTION OF
THE LADYZHENS KAYA-SMAGORINSKY SYSTEM

SUERLAN SILVA³⁰

This work addresses the Ladyzhenskaya-Smagorinsky differential turbulence model with partially known initial data. The main focus is the existence of insensitive controls in an arbitrary control domain, with respect to the local L^2 norm of the gradient of the solution to the model. The goal of this study is to find a control function that makes a functional dependent on the gradient of the state locally insensitive to perturbations in the initial data. The system in question presents both local and non-local nonlinearities, with the usual transport terms and turbulent viscosity, respectively. This problem can be reformulated as a non-standard null controllability problem, related to a nonlinear cascade system governed by a forward-time equation and a backward-time equation. To solve this problem, Carleman estimates, null controllability results, and new estimates for the solutions of the associated linear system are required. The main approach to establishing null controllability for the nonlinear cascade system relies on applying an inverse mapping theorem in infinite-dimensional spaces. Joint work with J. Límaco, J. Barreira and M. Hernández.

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BOUNDARY CONTROLLABILITY FOR THE BENJAMIN EQUATION POSED ON A
BOUNDED DOMAIN

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In this work we deal with the boundary controllability properties of the Benjamin equation posed on a bounded domain. We show that the equation is exactly controllable by controls that depend only on time and act on the boundary conditions of the system. Firstly, we prove that the exact controllability property is established for the linearized system through a method based on Fourier expansion of solutions of the adjoint system and Hilbert Uniqueness Method (H.U.M). For the nonlinear problem we study the problem by decomposing it into 2 problems: First the nonlinear problem with periodic conditions and initial data and second, the control of linearized problem with zero initial data; and studying each separately. This is joint work with Ademir Pastor.

References

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